Active Trailer Steering Control of an Articulated System With A Tractor and Three Full Trailers for Tractor-track Following

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Abstract: In this paper, we consider several control strategies to minimize off-tracking and rearward amplification of a multi-axle steering system with a tractor and three full trailers. A five-degree-of-freedom linear yaw-plane model is used to describe the vehicle dynamics. A tyre model describing the lateral tyre forces as a linear function of sideslip angles is incorporated in the yaw model. Given any arbitrary tractor track desired real-time by the driver, automated control inputs are the steering angles of front axles of the trailers. A minimum rearward amplification ratio (RWA), as a surrogate for minimum off-tracking, has been used as the control criterion for medium to high speeds to arrive at an optimal Linear Quadratic Regulator (LQR) controller. Robustness of the optimal controller with respect to tyre-parameter perturbations is then examined.

Based on the simulation results, we find that, active steering at all trailers gives the lowest RWA. To achieve acceptable levels of RWA and off-tracking, at least two of the three trailers must be actively steered. Among the three two-trailer-steering possibilities, actively steering trailers 1 and 2 is optimal and results in the lowest RWA with off-tracking practically eliminated. The optimal LQR controller is remarkably robust with respect to tyre-parameter variations.

Keywords: longer combination vehicle, LCV, heavy truck, off tracking, active trailer steering, rearward amplification, lateral dynamics and control.

Nomenclature

Tractor:

\[ c_1 \quad \text{Cornering stiffness of front wheel} \]
\[ c_2 \quad \text{Cornering stiffness of rear wheel} \]
\[ f_{yf} \quad \text{Lateral force in front wheel} \]
\[ f_{yr} \quad \text{Lateral force in rear wheel} \]
\[ f_{xf} \quad \text{Longitudinal force in front wheel} \]
\[ f_{x2} \quad \text{Longitudinal force in rear wheel} \]
\( I \) \( \) Principal yaw moment of inertia of the mass of tractor
\( m \) \( \) Total mass of tractors
\( S \) \( \) Distance between the front axle and mass centre
\( V \) \( \) Speed in the \( x \) direction
\( f \) \( \) Sideslip velocity
\( \delta \) \( \) Steering angle of the front wheel
\( \phi \) \( \) Rotation angle of tractor

Trailers: (1, 2, and 3 respectively)

\( c \) \( \) Cornering stiffness of front wheel
\( f \) \( \) Lateral forces in front wheels
\( l \) \( \) Lateral forces in rear wheels
\( s \) \( \) Longitudinal forces in front wheels
\( t \) \( \) Longitudinal forces in rear wheels
\( I \) \( \) Principle yaw moments of inertia
\( m \) \( \) Total masses of trailers
\( S \) \( \) Distances between the hitch and rear wheel of the preceding unit
\( r \) \( \) Distances between front axle and the preceding hitch
\( \delta \) \( \) Distances between front axle and mass centre of trailers
\( \delta \) \( \) Distances between mass centre and rear axles

Connecting hinges:

\( \psi \) \( \) Angle between tractor and trailer 1
\( \psi \) \( \) Angle between trailer 1 and trailer 2
\( \psi \) \( \) Angle between trailer 2 and trailer 3
\( F \) \( \) Coupling reaction forces in connecting point between tractor and trailer 1
\( F \) \( \) Coupling reaction forces in connecting point between trailer 1 and trailer 2
\( F \) \( \) Coupling reaction forces in connecting point between trailer 2 and trailer 3
1 Introduction

Expanded operations of a class of large trucks called “longer combination vehicles” (LCVs) may drastically improve the fuel, equipment and labor efficiencies of the trucking industry. However, LCV operations not only raise safety concerns for the surrounding traffic and the LCV drivers themselves but also can damage roadways and bridges not equipped to support the operations. A key source of safety and infrastructure issues for such operations is off-tracking, which has been used to refer to the general phenomenon that the rear wheels of a truck do not follow the track of the front wheels and wander off the travel lane. In this paper, off-tracking is defined as the maximum radial offset between the path of the tractor’s front-left wheel and that of trailer 3’s rear-left wheel during a 90-degree turn. Harkey et al. (1996) studied the operational characteristics of several configurations of LCV’s. Based on the policy on geometric design of highways and streets recommended by American Association of State Highway and Transportation Officials (AASHTO), they concluded that off-tracking experienced by LCVs while traveling on a freeway at high speeds can be accommodated by freeways designed according to the AASHTO policy, if the driver positions the tractor appropriately. Driver’s ability to position the tractor appropriately in turn requires low level of lateral oscillations.

Recently, Rangavajhula and Tsao (2005) studied the lateral dynamics of a tractor with up to three full trailers, developed a five-degree-of-freedom model, and demonstrated through numerical simulations that appropriate active steering of trailers can potentially overcome the problem of LCV off-tracking. For each of a set of tractor steering angles
selected for the purpose of a 90-degree turn, steering angles for the trailers that reduce
off-tracking significantly were chosen in an *ad hoc* fashion. Their study also shows that
for medium to high speeds, trailer steering has a significant effect on the trajectory of the
tractor and hence on that of the entire tractor-trailer system. Although the drastic
reduction in off-tracking and lateral oscillations achieved is indeed useful in terms of
showing the potential of trailer steering, it is unclear how the vehicle should be controlled
by the driver and by an automatic vehicle controller in order to achieve a desired vehicle
trajectory with minimum off-tracking and lateral oscillations.

A control strategy amenable for real-world application is to enable “tractor-track
following” by the all trailer wheels. In other words, trailer wheels should follow tractor
wheels for any arbitrary trajectory of the tractor desired by the driver. This enables the
driver to be concerned only about driving the tractor and positioning the tractor at or near
the lane center. Since all the trailers are designed to follow the trajectory of the tractor,
the driver can drive the entire tractor-trailer system by simply driving the tractor. In this
paper, we focus on such tractor-track-following control strategies and the corresponding
controller design, and seek to systematically reduce off-tracking with the approach of
optimal linear quadratic regulator (LQR).

It has been realized for some time that trailer steering is effective in reducing high- and
low-speed off-tracking, in reducing lateral forces and in increasing the lateral stability for
tractor-semi-trailer or tractor-and-one-full-trailer systems. However, only a small amount
of work exists in the control area about achieving the necessary steering control for
minimizing off-tracking and lateral forces of dynamic heavy vehicle systems with a tractor and multiple trailers. Aurell and Edlund (1989) investigated the effect of the location of steered axle of the semi-trailer on dynamic stability of the system. They showed that lateral acceleration and minimum turning radius could be reduced and off-tracking improved with additional steering axle. Wu and Lin (2003) showed that active multi-axle steering for a tractor-and-one-full-trailer system could improve lateral stability characteristics, reduce lateral loads, and provide better manoeuvrability of the system. Chen and Tomizuka (2000), in their study on lateral control of tractor-semi-trailer-type commercial vehicles, showed that by applying coordinated braking, trailer yaw velocities could be reduced. Palcovics and El-Gindy (1995, 1996) considered several control strategies with active unilateral braking (i.e., differential or individual braking) for tractor-semi-trailer system and demonstrated that active braking control at the lead unit improved roll stability, and the controller so designed was robust and insensitive to vehicle parameters. They encountered a situation similar to what Rangavajhula and Tsao (2005) found for trailer steering, namely strong influence of the trailer controller on the tractor trajectory. We note, in particular, the later work of El-Gindy et al. (1998, 2001) in which they overcome this problem by applying an active control moment at the dolly without affecting driver controls.

Unsafe operating conditions of LCVs may manifest through off-tracking and through undesirable rolling as well as yawing motions at critical conditions. In addition, rearward amplification, quantified with the rearward amplification ratio (RWA) defined by El-Gindy et al. (2001), also impacts the roll stability of articulated systems. In their paper,
RWA was defined as the ratio of the peak lateral accelerations at the rearmost trailer’s centre of gravity (CG) and the powered unit’s (tractor) CG during a lane-change manoeuvre. They used RWA as a control criterion in the design of vehicle-handling controller. Their simulation results show that the RWA can be reduced significantly without significant changes to the uncontrolled vehicle’s trajectory when active yaw torque is applied to the trailer unit. In this paper, we adapt their idea of using the ratio of the two peak lateral accelerations to measure rearward amplification and define RWA as “the ratio of peak lateral acceleration at trailer 3’s CG to the peak lateral acceleration at tractor’s CG”. Rolling motions are neglected, and the focus is on the off-tracking performance and reduction of lateral oscillations of the controlled system.

At medium- to high-speed operations, LCV’s, due to the availability of several lateral degrees of freedom, are susceptible to large lateral forces that result in large lateral oscillations, off tracking, and internal resonances, which may lead to loss of stability. In this paper, we focus on medium to high speeds. At these speeds, since large lateral forces constitute a major underlying reason for both off-tracking and rearward amplification, we expect to find significant correlation between RWA and off-tracking. Therefore, we seek to reduce significantly or eliminate off-tracking by systematically minimizing RWA. In other words, we use RWA as a surrogate measure of the degree of off-tracking. In addition, a focus on minimizing RWA will result in reduced amplitudes of lateral oscillations. Another reason for this approach is explained as follows.
We are interested in a good tractor-track following performance for any set of arbitrary driver steering inputs and hence for any arbitrary tractor track. Note that a precise mathematical definition of off-tracking for such arbitrary tractor track may be a complex task because of the need to define the maximum deviation of a trailer’s trajectory with respect to the arbitrary and possibly erratic trajectory of the tractor by comparing two erratic continuous curves related by vehicle dynamics through time. Therefore a direct minimization of off-tracking for all possible tractor tracks will be more complex. This is contrasted with the simple definition of RWA as a ratio between two measures, each of which can be obtained by observing one (continuous) time-series - the lateral acceleration of one moving unit through time. These considerations provide the motivation for us to follow El-Gindy et al. (2001) in adopting and adapting their idea of rearward amplification ratio (RWA) as a major performance criterion in our controller design. The use of RWA as a surrogate measure for off-tracking may then alleviate the need for a specialized treatment and control strategy for removal/reduction of off-tracking.

Equipping trailers with either trailer steering or differential braking incurs equipment and perhaps retrofit costs. To reduce the need for extra equipment in actual implementations, we study the effect of several different combinations of the locations of active trailer steering control to arrive at a most cost-effective number of control inputs and a set of cost-effective locations to achieve off-tracking reduction and other desired lateral-performance characteristics.
We adopt the approach of Linear Quadratic Regulator used in by El-Gindy et al. (2001), in which an integral equation involving a vehicle-state vector, trailer-steering control-input vector, and two sets of weights used to reflect user preferences is defined as the objective for minimization. Tyre parameters are included in the controller design explicitly. The trailer-steering control inputs are characterized by a gain vector as the variables, and the weights are chosen to minimize RWA. RWA is optimized for a sinusoidal driver steering input, corresponding to a four-second lane-change manoeuvre, and the corresponding gain vector is chosen to form the core of the control algorithm, for actual implementation and computer simulation. We also study the reduction of off-tracking resulting from the controller optimized for the four-second lane change with respect to the RWA. RWA performance is also studied for sinusoidal frequencies corresponding to two- and eight-second lane changes performed with the controller optimized for a four-second lane change.

This paper is organized as follows. Section 2 describes the linear yaw-plane vehicle model used in the design of control strategies. Tyre model is included in the vehicle model and thus explicitly influences the controller design. In section 3, design of the LQR controller is presented. In section 4 we discuss the vehicle response to active steering control. In Section 5, robustness of the controller is examined by perturbing the tyre cornering stiffness parameters. Conclusions follow in Section 6.
2 Linear Yaw Plane Model

Rangvajhula and Tsao (2005) extended the yaw model developed by Wu and Lin (2003) for one tractor and one trailer to include up to two additional trailers. In this paper, we adopt the extended model and also the dimension data assumed in both papers for the tractor, the full trailer and their connection. For Wu and Lin’s tractor-and-one-trailer problem, there are three degrees of freedom corresponding to side slip velocity, yawing motion of the tractor and trailer, and the articulation angle between the tractor and the trailer. For a tractor-and-three-trailers system considered in this paper, the articulation angles between trailers 1 and 2 and between trailers 2 and 3 provide two additional degrees of freedom, resulting in a five-degree-of-freedom motion. As mentioned earlier, only the front axle of a trailer is equipped with steering capability. Tractor and trailer units are considered as rigid masses with body-fixed coordinates centered on the mass centers of the units. Aerodynamic forces, rolling and pitching motions, braking inputs and the longitudinal forces generated by tires and road interactions are neglected. Lateral forces generated by tires and road interactions, which constitute the main influence on yaw dynamics and lateral stability, are included in the model. A schematic model of the articulated system is sketched in Figure 1. Each wheel in this zero-vehicle-width model represents an axle.

Using Newton’s Second Law, equations of motion are derived for the tractor and trailer units. These equations are linearized under the small-displacements assumption. Further simplification is achieved by assuming steady forward speed. Applying Newton’s
Second Law, the equations of motion for the tractor in $x - y$ body-fixed moving-coordinates are given as:

\begin{align*}
m_i(\dot{V}_x - \omega V_y) &= F_{x_i} - f_{yf} \sin(\delta_f) \\
m_i(\dot{V}_x + \omega V_y) &= -F_{y_i} + f_{yf} + f_{yf} \cos(\delta_f) \\
I_i \dot{\omega} &= dF_{y_i} - f_{yr} S_2 + f_{yf} S_1 \cos(\delta_f) \tag{1}
\end{align*}

where $\omega = \dot{\phi}$, the dot ($\dot{}$) denotes $\frac{d}{dt}$, and the constant $d$ is given in part 1 of the Appendix.

The equations of motion for trailer 1 in $x' - y'$ coordinates are given as:

\begin{align*}
m_2(\dot{V}_{x'} - V_{y'} \omega_1) &= F_{x_2} - F_{x_{y_1}} \cos(\psi_1) - F_{y_{y_1}} \sin(\psi_1) - f_{y_1} \sin(\delta_{r_1}) \\
m_2(\dot{V}_{y'} + V_{x'} \omega_1) &= -F_{y_2} + F_{y_{y_1}} \cos(\psi_1) + f_{x_2} + f_{y_1} \cos(\delta_{r_1}) \\
I_2 \dot{\omega}_1 &= lF_{y_2} - f_{y_1} S_6 + eF_{y_{y_1}} \cos(\psi_1) - eF_{y_1} \sin(\psi_1) + f_{y_1} S_5 \cos(\delta_{r_1}) \tag{2}
\end{align*}

where $\omega_1 = \omega - \psi_1$, and $e$ and $l$ are given in part 1 of the Appendix. Similarly, equations of motion for trailer-2 and trailer-3 in $x'' - y''$ and $x''' - y'''$ coordinates, given by Equations (3) and (4) respectively are as follows:

\begin{align*}
m_3(\dot{V}_{x''} - V_{y''} \omega_2) &= F_{x_3} - F_{x_{y_2}} \cos(\psi_2) - F_{y_{y_2}} \sin(\psi_2) - f_{y_2} \sin(\delta_{r_2}) \\
m_3(\dot{V}_{y''} + V_{x''} \omega_2) &= -F_{y_3} + f_{y_2} + f_{y_2} \cos(\delta_{r_2}) + F_{y_{y_2}} \cos(\psi_2) - F_{y_2} \sin(\psi_2) \\
I_3 \dot{\omega}_2 &= nF_{y_3} - f_{y_3} S_10 + mF_{y_2} \cos(\psi_2) - mF_{y_2} \sin(\psi_2) + f_{y_2} S_9 \cos(\delta_{r_2}) \tag{3}
\end{align*}

and

\begin{align*}
m_4(\dot{V}_{x'''} - V_{y'''} \omega_3) &= -F_{x_4} \cos(\psi_3) - F_{y_{y_3}} \sin(\psi_3) - f_{y_3} \sin(\delta_{r_3}) \\
m_4(\dot{V}_{y'''} + V_{x'''} \omega_3) &= f_{x_4} + f_{y_3} \cos(\delta_{r_3}) + F_{y_{y_3}} \cos(\psi_3) - F_{y_3} \sin(\psi_3) \\
I_4 \dot{\omega}_3 &= -f_{y_3} S_14 + qF_{y_3} \cos(\psi_3) - qF_{y_3} \sin(\psi_3) + f_{y_3} S_{13} \cos(\delta_{r_3}) \tag{4}
\end{align*}

where $m, n$ and $q$ are given in part 1 of the Appendix. Here

\begin{align*}
\omega_2 &= \omega - \psi_1 - \psi_2, \quad \omega_3 = \omega - \psi_1 - \psi_2 - \psi_3.
\end{align*}
Since the lateral tire force is the major influence on the yaw dynamics and stability of the vehicle, we assume that the longitudinal forces denoted by \( f_i \) \(( \text{for } i = 1,8)\) at tire \( i \) are zero. We further assume that the tractor steering angle \( \delta_f \) and the articulation angles \((\psi_1, \psi_2, \text{and } \psi_3)\) to be small.

Eliminating the coupling reactions \( F_{xi}, F_{yi} \) \(( \text{for } i = 1,3)\) from Equations (1) - (4), we obtain a five-degrees-of-freedom model for the tractor and the three trailers. Writing this in first order form we have:

\[
[A]{\dot{p}(t)} = [B]{p(t)} + [L]\delta_f + [C_\delta]u,
\]

where \( p(t) \) is the state vector and \( u \) is the control vector. They are given as:

\[
p(t) = [\dot{\phi} \psi_1 \psi_2 \psi_3 \psi_1 \psi_2 \psi_3]^T
\]

\[
u = [\delta r_1, \delta r_2, \delta r_3]^T.
\]

\( \{L\} \) is a \( 5 \times 1 \) column vector associated with steering input/force. Elements of matrices \( [A], [B], \{L\} \) and \( [C_\delta] \) are given in part 1 of Appendix. Equation (5) can further be reduced to state-space form as:

\[
\dot{p}(t) = [A_1]{p(t)} + [B_1]\delta_f + [B_2]u,
\]

where \([A_1] = [A]^{-1}[B], [B_1] = [A]^{-1}[L] \) and \([B_2] = [A]^{-1}[C_\delta] \).

### 3 LQR Controller Design

The objective of our controller design is to drastically reduce off-tracking in the system by reducing RWA while improving other dynamic performance of the system. To
minimize equipment cost, it is important to study whether all three trailers must be so equipped and, if not, which of the three trailers should be equipped for satisfactory reduction of off-tracking. As mentioned earlier, a total of seven cases were examined to arrive at the most cost-effective controller design. Our ultimate objective is to arrive at a minimum number of control inputs at optimal locations that will result in acceptable levels of off-tracking and RWA. For each of the seven cases, we devise an optimal feedback controller. In the design of the optimal feedback controller, we minimize the performance index $J$ of the vehicle system given by:

$$J = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{\infty} (p^T Q_o p + u^T R_o u) dt,$$

where $p$, $u$, $Q_o$, and $R_o$ are the state vector, the control-input vector, state weighting matrix and control weighting matrix respectively. Note that $Q_o$ (positive semi-definite) and $R_o$ (positive definite) are designer-selected matrices. They and the other design parameters, including the data about the tractor and trailers, for the optimal LQR controller are given in part 2 of the Appendix. Note that, for each of the seven cases, such an optimal controller corresponds to the selected $Q_o$ and $R_o$, and results in a particular corresponding RWA value. We seek among all such optimal controllers, i.e., among all possible $Q_o$ and $R_o$, the one that produces the least RWA for the sinusoidal driver steering input corresponding to a four-second lane-change manoeuvre at the forward speed of 50 ft/sec. (34 miles/hour). We then study the off-tracking performance of this least-RWA optimal controller during a 90-degree turn. The control input can be determined by solving the algebraic Ricatti equation for $P^*$ given by:
\[ P^*A_1 + A_1^T P^* - P^* B_2 R_0^{-1} B_2 P^* + Q_0 = 0 \]

The full state feedback control law is expressed as:

\[
u = -K_{opt} \ p
\]

The optimal feedback gain matrix \( K_{opt} \) that minimizes the performance index \( J \) is in the form of

\[
K_{opt} = R_0^{-1} B_2^T P^*
\]

4 Vehicle Response to Active Steering Control

In this section, we study the performance of the vehicle under different control strategies. As mentioned earlier, the forward velocity of the vehicle is set to 50 ft/sec (≈ 34 miles/hr) for all the cases simulated in this paper, and all the vehicle data are given in part 3 of the Appendix. A linear stability analysis by Rangavajhula and Tsao (2005) showed that this articulated tractor-and-three-trailer system traveling at 50 ft/sec without active trailer steering has a pair of lightly damped poles. Although the system is stable, perturbations take a long time to decay. (As will become clear later, active trailer steering will enable a quick decay.) The trajectories of a tractor and the three non-steered trailers for a sharp turn of radius of 450 ft (at driver steering angle of 0.005 \( \pi \)), those for a four-second lane-change manoeuvre, and the lateral accelerations at the centers of gravity of the tractor and trailer 3 are shown in Figures 2(a)-2(c) respectively. RWA for this case is 4.4 as shown in Table 1. We note that, for the tractor-and-three-full-trailer vehicle configuration, lateral acceleration at center of gravity (CG) of trailer 3 during a turn of a radius of approximately 450 ft at a forward speed of 50 ft/sec is 10.5 ft/sec\(^2\) (0.32 g), and this
acceleration is comparable to the acceleration experienced by the trailer 3 of this vehicle system during the four-second lane-change manoeuvre. This explains why the controller designed to minimize RWA performs well in off-tracking reduction during a 90-degree turn.

For the cases of active steering control only at one of the three trailers, the minimum RWA is 2.43 (a decrease of about 55%), corresponding to case where only trailer 1 is actively steered. Active steering only at trailer 2 or only at trailer 3 does not reduce the RWA or off-tracking. Figure 3 reveals vehicle behaviors corresponding to active steering at trailer 1. Figure 3(a) shows the off-tracking performance for a range of tractor steering angles; even though RWA is reduced by about 55% and off-tracking is reduced to about 3 ft (for turn radius of 450 ft), lightly damped modes continue to exist as shown in the Figures 3(b, c). Table 1 summarizes the optimal values of RWA for all seven control cases considered and the corresponding off-tracking values. Acceptable level of off-tracking values are observed for RWA between 0.68 - 1.00. Active steering at trailers 1 and 2 gives the most cost-effective performance with a RWA value of 0.74 (83% reduction from uncontrolled case) and off-tracking of 0.16 ft (98% reduction from uncontrolled case).

Figures 4 and 5 describe vehicle behaviors corresponding to the case of active steering for trailers 1 and 2 and the case of all-trailer active steering, respectively. As shown in Figure 4(a) and Figure 5(a) for these cases, the rear wheels of trailer 3 follow the tractor’s track (front wheels) accurately for several steering angles (driver input) of the tractor’s
front wheels. Lateral oscillations of trailers during a lane change (Figures 4(b), 5(b)) are reduced significantly (as compared to uncontrolled case as shown in Figure 2(b)), and there is a 60% reduction in lateral forces as shown in Figures 4(c) and 5(c).

Based on Figures 4 and 5, it is visually clear that steering trailer 1 and trailer 2 leads to virtual elimination of off-tracking. In Table 2, off-tracking for several angles of driver steering for this case is presented. We see satisfactory tractor-track-following characteristics; off-tracking is practically eliminated for all the driver inputs. From Figure 3, it is also clear that steering only one trailer is not adequate for a significant reduction of off-tracking. Based on the vehicle response to seven combinations of the locations of active steering control inputs, we arrive at the minimum number of two active steering inputs with trailers 1 and 2 as the optimal locations, for a satisfactory off-tracking reduction. We treat this combination as the most cost-effective one; the controller designed for this case has the low RWA of 0.74 and virtually eliminated off-tracking, for a four-second lane change. Note that, in what follows, we focus on this particular case.

To ascertain the robustness of this optimal LQR controller devised explicitly for a four-second lane change with respect to faster or slower lane changes, we study the performance of this controller during a two-second and eight-second lane-change manoeuvre. The performance is shown in Figures 6 and 7, respectively. For the uncontrolled case, lateral accelerations experienced by the trailer 3 (at CG) are very high (≈ 0.5 g) for a two-second lane change and very low (≈ 0.06 g) for an eight-second lane change. In both cases, significant reduction in RWA and reduction in lateral forces
experienced by the trailers is achieved. Lateral oscillations are completely damped as seen in Figures 6(c) and 7(c).

The controller designed for active steering control has a considerable influence on the tractor’s path. A feed-forward gain may be introduced to overcome this nuisance phenomenon of trailer steering. Alternatively, since, for arbitrary driver steering inputs, all the trailers’ rear wheels have accurate tractor-track-following characteristics, driver input through intuition may suffice for the necessary tractor steering. Such driver interaction would be difficult if trailers’ wheels do not track the tractor-track accurately.

In the next section we examine the robustness of the designed controller for this most cost-effective location combination for trailer steering with respect to tyre-parameter variations.

5 Controller Sensitivity to Tyre Parameter Variations

In this section, we examine the robustness of the optimal controller designed in the previous sections for active steering at trailers 1 and 2. Tyre cornering stiffness corresponding to selected axles are the only parameters that are being perturbed. This approach is the same as the one presented by El–Gindy et al. (2001) in the study of robustness of their controller to tyre parameter variations. Tyre model has been explicitly incorporated into our vehicle model. (The elements of the matrix $[C_\delta]$ in Equation 5 are functions of the cornering stiffness and thus are included in the design of the controller.) RWA variations and trajectories of the tractor and rearmost wheels (of trailer 3) are examined when the cornering stiffness of the tyres are independently varied by $\pm 20\%$ at
each axle. RWA ratios corresponding to independent cornering stiffness variations at each axle are shown in Figure 8(a). Percentage error in RWA (compared to unperturbed RWA of 0.74) is presented in Table 3. Largest variation of 0.94% in RWA (off-tracking ≈ 0.17 ft) is observed at axle 3 followed by a variation of 0.81% in RWA (off-tracking ≈ 0.25 ft) at axle 5; they correspond to independent variations of cornering stiffness of the front wheels of trailer 1 and trailer 2, respectively. The trajectories of the tractor and the actively steered trailer 3 resulting from parameter variations for the front axle of the tractor are shown in Figure 8(b). Figure 8(c) shows the same trajectories resulting from independent parameter variations for trailer 1 and trailer 2. These trajectories produced with the ±20% parameter variations for both the cases are indistinguishable from their nominal counterparts as shown in Figure 4(b). The above results indicate that the controller is remarkably robust with respect to tyre parameter variations. This robustness is a direct consequence, as suggested by El-Gindy et al. (2001), of including the tyre parameters in the design of the controller.

6 Conclusions

In this paper, we studied several control strategies to reduce off-tracking in an articulated system with a tractor and three full trailers. A five-degree-of-freedom yaw model with an embedded tyre model was employed to study the performance of the designed controller. A linear quadratic regulator (LQR) controller was designed following the approach of El-Gindy et al. (2001), based on the objectives of minimizing RWA and accurate tractor-track following. Based on the simulation results, we find that, to achieve acceptable levels of RWA and off-tracking with trailer-steering, at least two of the three trailers must
be actively controlled. The most cost-effective design corresponds to a controller designed for active steering at trailer 1 and trailer 2. The LQR controller for this case is remarkably robust to tyre parameter variations. With this most cost-economic design, RWA is substantially reduced and off-tracking is practically eliminated. This is a particularly attractive feature, since, with the aid of this controller, the rearmost wheels of the trailers can potentially follow the tractor’s track for a wide range of and arbitrary driver inputs, and hence the controller addresses some key safety and infrastructure concerns associated with LCV operations.
References


Appendix

Part 1:

\[
\{d = (S_2 + S_3), e = (S_4 + S_5), l = (S_6 + S_7), m = (S_8 + S_9), n = (S_{10} + S_{11}), q = (S_{12} + S_{13})\}
\]

\[
[A] = \begin{bmatrix}
m_{11} & m_{12} & m_{13} & m_{14} & m_{15} & 0 & 0 & 0 \\
m_{21} & m_{22} & m_{23} & m_{24} & m_{25} & 0 & 0 & 0 \\
m_{31} & m_{32} & m_{33} & m_{34} & m_{35} & 0 & 0 & 0 \\
m_{41} & m_{42} & m_{43} & m_{44} & m_{45} & 0 & 0 & 0 \\
m_{51} & m_{52} & m_{53} & m_{54} & m_{55} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}, \quad [B] = \begin{bmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & k_{13} & k_{14} & k_{15} \\
c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & k_{23} & k_{24} & k_{25} \\
c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & k_{33} & k_{34} & k_{35} \\
c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & k_{43} & k_{44} & k_{45} \\
c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & k_{53} & k_{54} & k_{55} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
[L]^T = \{L_{45}, L_{25}, L_{55}, L_{45}, L_{55}, 0, 0, 0\}, \quad [C_8] = \begin{bmatrix}
\mathbf{u}_{11} & \mathbf{u}_{12} & \mathbf{u}_{13} \\
\mathbf{u}_{21} & \mathbf{u}_{22} & \mathbf{u}_{23} \\
\mathbf{u}_{31} & \mathbf{u}_{32} & \mathbf{u}_{33} \\
\mathbf{u}_{41} & \mathbf{u}_{42} & \mathbf{u}_{43} \\
\mathbf{u}_{51} & \mathbf{u}_{52} & \mathbf{u}_{53} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

Where:

\[
\begin{align*}
m_{11} &= m_1 + m_2 + m_3 + m_4 \\
m_{12} &= -[m_2(d + e) + m_3(d + e + l + m) + m_4(d + e + l + m + n + q)] \\
m_{13} &= [m_3 e + m_4(e + l + m) + m_4(e + l + m + n + q)] \\
m_{14} &= [m m_3 + m_4(m + n + q)] \\
m_{15} &= m_4 q \\
m_{21} &= -d(m_2 + m_3 + m_4) \\
m_{22} &= [d m_2(d + e) + d m_3(d + e + l + m) + d m_4(d + e + l + m + n + q) + I_1] \\
m_{23} &= -[d m_2 e + d m_3(e + l + m) + d m_4(e + l + m + n + q)] \\
m_{24} &= d[m m_2 + m_4(e + l + m + n + q)] \\
m_{25} &= d m_4 q
\end{align*}
\]
\[
\begin{align*}
    m_{31} &= -(l+e)(m_3 + m_4) + m_2 e \\
    m_{32} &= [(l+e)(m_3 + m_4) + m_2 e(d+e) + I_2] \\
    m_{33} &= -(l+e)(m_3(e + l + m) + m_4(e + l + m + n + q)) \mp m_2 e^2 + I_2 \\
    m_{34} &= -(l+e)(m_3 + m_4(m + n + q)) \\
    m_{35} &= -m_4 q(l+e) \\
    m_{41} &= -[m_3 + (m + n)m_3] \\
    m_{42} &= [m_3(d + e + l + m) + (m + n)(d + e + l + m + n + q)m_4 + I_3] \\
    m_{43} &= -[m_3(e + l + m) + m_4(e + l + m + n + q) + I_5] \\
    m_{44} &= -[m^2 m_3 + (m + n + q)(m + n)m_4 + I_3] \\
    m_{45} &= -(m + n)m_4 q \\
    m_{51} &= -qm_4 \\
    m_{52} &= [qm_4(d + e + l + m + n + q) + I_4] \\
    m_{53} &= -[qm_4(e + l + m + n + q) + I_4] \\
    m_{54} &= -[qm_4(m + n + q) + I_4] \\
    m_{55} &= -(q^2 m_4 + I_4)
\end{align*}
\]

\[
\begin{align*}
    c_{11} &= -(c_1 + c_2 + c_3 + c_4 + c_5 + c_6 + c_7 + c_8) / V_x \\
    c_{12} &= [(m_1 + m_2 + m_3 + m_4)V_x - (c_1 S_1 - c_2 S_2 - c_3 (d + S_4) - c_4 (d + e + S_5) - c_6 (d + e + l + S_6) \\
    & \quad - c_8 (d + e + l + m + S_{10}) - c_7 (d + e + l + m + n + S_{12}) - c_9 (d + e + l + m + n + q + S_{14})) / V_x \} \\
    c_{13} &= -(c_3 S_4 + c_4 (e + S_5) + c_5 (e + l + S_6) + c_6 (e + l + m + S_{10}) + c_7 (e + l + m + n + S_{12}) \\
    & \quad + c_8 (e + l + m + n + q + S_{14})) / V_x \\
    c_{14} &= -(c_7 S_1 + c_8 (q + S_{14})) / V_x \\
    c_{15} &= -(c_7 S_1 + c_8 (q + S_{14})) / V_x \\
    c_{21} &= -(c_2 S_2 - c_1 S_3 + d(c_3 + c_4 + c_5 + c_6 + c_7 + c_8)) / V_x \\
    c_{22} &= -(d(m_1 + m_2 + m_3 + m_4)V_x + (c_1 S_1^2 + c_2 S_2^2 + d c_3 (d + S_4) + d c_4 (d + e + S_5) + d c_5 (d + e + l + S_6) \\
    & \quad + d c_6 (d + e + l + m + S_{10}) + d c_7 (d + e + l + m + n + S_{12}) + d c_9 (d + e + l + m + n + q + S_{14})) / V_x \} \\
    c_{23} &= d(c_3 S_4 + c_4 (e + S_5) + c_5 (e + l + S_6) + c_6 (e + l + m + S_{10}) + c_7 (e + l + m + n + S_{12}) \\
    & \quad + c_8 (e + l + m + n + q + S_{14})) / V_x \\
    c_{24} &= d(c_7 S_1 + c_8 (q + S_{14})) / V_x \\
    c_{25} &= d(c_7 S_1 + c_8 (q + S_{14})) / V_x
\end{align*}
\]
\[
\begin{align*}
  c_{41} &= \{S_8c_5 + c_6(m + S_{10}) + (m + n)(c_7 + c_8)\} / V_x \\
  c_{42} &= -\{(m_3 + m_4)(l + e) + m_7e\} V_x + \{c_5S_4(d + S_4) + c_4(e + S_6) + (e + l)c_5(d + e + l + S_8) \\
  &\quad + c_6(d + e + l + m + n + S_{12}) + c_7(d + e + l + m + n + q + S_{14})\} / V_x \\
  c_{43} &= \{S_8c_5(e + l + S_8) + c_6(m + S_{10})(e + l + m + S_{10}) + (m + n)(c_7(e + l + m + n + S_{12}) \\
  &\quad + c_8(e + l + m + n + q + S_{14}))\} / V_x \\
  c_{44} &= \{c_6S_8^2 + c_6(m + S_{12})^2 + (m + n)(c_7(m + n + S_{12}) + c_8(m + n + q + S_{14}))\} / V_x \\
  c_{45} &= \{(m + n)(c_7S_{12} + c_8(q + S_{14}))\} / V_x \\
  c_{51} &= \{S_{12}c_7 + (q + S_{14})c_8\} / V_x \\
  c_{52} &= -\{qm_4V_x + c_7S_{12}(d + e + l + m + n + S_{12}) + (q + S_{14})(d + e + l + m + n + q + S_{14})\} / V_x \\
  c_{53} &= \{c_7S_{12}(e + l + m + n + S_{12}) + c_8(q + S_{14})(e + l + m + n + q + S_{14})\} / V_x \\
  c_{54} &= \{S_{12}c_7(m + n + S_{12}) + c_8(q + S_{14})(e + l + m + n + q + S_{14})\} / V_x \\
  c_{55} &= \{c_7S_{12}^2 + c_8(q + S_{14})^2\} / V_x \\
  c_{31} &= \{(e + l)(c_5 + c_6 + c_7 + c_8) + c_4(e + S_6) + c_3S_4\} / V_x \\
  c_{32} &= -\{(m_3 + m_4)(l + e) + m_7e\} V_x + \{c_5S_4(d + S_4) + c_4(e + S_6) + (e + l)c_5(d + e + l + S_8) \\
  &\quad + c_6(d + e + l + m + n + S_{12}) + c_7(d + e + l + m + n + q + S_{14})\} / V_x \\
  c_{33} &= \{c_6(e + S_6)^2 + c_3S_4^2 + (e + l)(c_5(e + l + S_8) + c_6(e + l + m + S_{10}) + c_7(e + l + m + n + S_{12}) \\
  &\quad + c_8(d + e + l + m + n + q + S_{14}))\} / V_x \\
  c_{34} &= \{(e + l)(c_5S_8 + c_6(m + S_{10}) + c_7(m + n + S_{12}) + c_8(m + n + q + S_{14}))\} / V_x \\
  c_{35} &= \{(e + l)(c_7S_{12} + c_8(q + S_{14}))\} / V_x \\
  k_{11} &= k_{12} = k_{21} = k_{22} = k_{31} = k_{32} = k_{41} = k_{42} = k_{51} = k_{52} = 0 \\
  k_{13} &= -(c_3 + c_4 + c_5 + c_6 + c_7 + c_8) \\
  k_{14} &= -(c_5 + c_6 + c_7 + c_8) \\
  k_{15} &= -(c_7 + c_8) \\
  k_{23} &= d(c_3 + c_4 + c_5 + c_6 + c_7 + c_8) \\
  k_{24} &= d(c_4 + c_5 + c_7 + c_8) \\
  k_{25} &= d(c_7 + c_8) \\
  k_{33} &= \{(e + l)(c_5 + c_6 + c_7 + c_8) + c_4(e + S_6) + c_3S_4\} \\
  k_{34} &= (e + l)(c_6 + c_7 + c_8 + c_3) \\
  k_{35} &= (e + l)(c_7 + c_8) \\
  k_{43} &= \{S_8c_5 + c_6(m + S_{10}) + (m + n)(c_7 + c_8)\} \\
  k_{44} &= \{S_8c_5 + c_6(m + S_{10}) + (m + n)(c_7 + c_8)\} \\
  k_{45} &= (m + n)(c_7 + c_8)
\end{align*}
\]
\[ \{ k_{33} = k_{44} = k_{55} = \{ S_1 c_7 + (q + S_{14}) c_8 \} \] \\
\[ \{ L_{45} = -c_1, L_{23} = -c_1 S_1, L_{35} = 0, L_{45} = 0, L_{53} = 0 \] \\
\[ \begin{cases} 
  u_{11} = -c_3, u_{21} = c_3 d, u_{31} = c_3 s_4, u_{41} = 0, u_{51} = 0 \\
  u_{12} = -c_4, u_{22} = c_4 d, u_{32} = c_4 (e + l), u_{42} = c_4 s_5, u_{52} = 0 \\
  u_{13} = -c_7, u_{23} = c_7 d, u_{33} = c_7 (e + l), u_{43} = c_7 (m + n), u_{53} = c_7 s_{12} 
\end{cases} \]

**Part 2:**

Data for tractor and trailer units (all trailer units considered in this paper are identical)

**Tractor:**
- Weight, loaded = 15,000 lb;
- \( I_1 = 16,000 \) lb-ft-sec\(^2\)
- \( S_1 = 8.4 \) ft
- \( S_2 = 3.6 \) ft
- \( S_3 = 3.0 \) ft
- \( c_1 = -14,766 \) lb / rad
- \( c_2 = -29,532 \) lb / rad

**Trailer Units:**
- Weight, loaded = 12,000 lb
- \( I_1 = I_2 = I_3 = 9,250 \) lb-ft-sec\(^2\)
- \( S_4 = S_8 = S_{12} = 8.0 \) ft
- \( S_5 = S_9 = S_{13} = 5.5 \) ft
- \( S_6 = S_{10} = S_{14} = 4.5 \) ft
- \( c_3 = c_4 = c_5 = c_6 = c_7 = c_8 = -17,089 \) lb / rad

\[ Q_0 = \text{diag} (2, 2, 2, 2, 2, 2) \]

\[ R_0 = \text{diag} (0.04, 0.04, 0.04) \]
Figure 1  Linear Yaw plane model for a system of a tractor and 3 trailers.

Figure 2  Uncontrolled System: (a) 90 degree turn trajectories of tractor front and trailer 3 rear wheel for $\delta_f = 0.005\pi$ (b) Lane Change Manoeuvre, (c) Lateral accelerations at tractor and Trailer 3 centers’ of gravity.

Figure 3  Active steering control at trailer 1: (a) 90-degree turn trajectories of tractor front and trailer 3 rear wheel for several values of $\delta_f$ (b) Lane Change Manoeuvre, (c) Lateral accelerations at tractor and trailer 3 centers’ of gravity.

Figure 4  Active steering control at trailers 1 and 2: (a) 90-degree turn trajectories of tractor front and trailer 3 rear wheels for several values of $\delta_f$ (b) Lane Change Manoeuvre, (c) Lateral accelerations at tractor and Trailer 3 centers’ of gravity.

Figure 5  Active steering control at all trailers: (a) 90-degree turn trajectories of tractor front and trailer rear wheels for several values of $\delta_f$ (b) Lane Change Manoeuvre, (c) Lateral accelerations at tractor and Trailer 3 centers’ of gravity.

Figure 6  2-second Lane-Change Manoeuvre: (a) Uncontrolled trajectories of tractor front and trailer 3 rear wheels (b) Trajectories of tractor front and trailer 3 rear wheels with active steering at trailer 1 and trailer 2 and with optimum controller (c) Lateral accelerations at tractor and trailer 3 centers’ of gravity for the controlled case.

Figure 7  8-second Lane-Change Manoeuvre: (a) Uncontrolled trajectories of tractor front and trailer 3 rear wheels (b) Trajectories of tractor front and trailer 3 rear wheels with active steering at trailer 1 and trailer 2 and with optimum controller (c) Lateral accelerations at tractor and trailer 3 centers’ of gravity for the controlled case.

Figure 8  Actively controlled steering at trailer 1 and trailer 2: (a) RWA for axle parameter variation (b) Trajectories of tractor front and trailer 3 rear wheels for steering axle parameter variation (c) Trajectories of tractor front and trailer 3 rear wheels for trailer 1 axle parameter variation and trailer 2 axle parameter variation (same result is observed for trailer 1 and trailer 2 variation as well as for +20% and -20% variation).
Tables

**Table 1**: Minimum RWA for the Seven Steering Location Combinations & the Corresponding Off-Tracking Performance

<table>
<thead>
<tr>
<th>Control Case</th>
<th>RWA</th>
<th>Off-Tracking (ft) (Turn Radius of 450 ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncontrolled</td>
<td>4.4</td>
<td>7.00</td>
</tr>
<tr>
<td>Active steering at trailer 1</td>
<td>2.43</td>
<td>3.30</td>
</tr>
<tr>
<td>Active steering at trailer 2</td>
<td>4.14</td>
<td>&gt; 7.00</td>
</tr>
<tr>
<td>Active steering at trailer 3</td>
<td>&gt; 4.4</td>
<td>&gt; 7.00</td>
</tr>
<tr>
<td><strong>Active steering at trailers 1 &amp; 2</strong></td>
<td>0.74</td>
<td><strong>0.16</strong></td>
</tr>
<tr>
<td>Active steering at trailers 1 &amp; 3</td>
<td>0.86</td>
<td>1.65</td>
</tr>
<tr>
<td>Active steering at trailers 2 &amp; 3</td>
<td>1.23</td>
<td>3.12</td>
</tr>
<tr>
<td>Active steering at all trailers</td>
<td>0.68</td>
<td>0.75</td>
</tr>
</tbody>
</table>

**Table 2**: Off-tracking at several driver steering angles for active steering at trailer 1 and 2

<table>
<thead>
<tr>
<th>Steering Angle (Tractor)</th>
<th>Off-Tracking (ft) (450 ft turn radius)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005π</td>
<td>0.03</td>
</tr>
<tr>
<td>0.01π</td>
<td>0.10</td>
</tr>
<tr>
<td>0.02π</td>
<td>0.12</td>
</tr>
<tr>
<td>0.03π</td>
<td>0.16</td>
</tr>
<tr>
<td>0.04π</td>
<td>0.19</td>
</tr>
<tr>
<td>0.05π</td>
<td>0.21</td>
</tr>
</tbody>
</table>

**Table 3**: Percentage error in RWA due to tyre parameter variations for active steering at trailer 1 and 2

<table>
<thead>
<tr>
<th>Axle</th>
<th>+20% nominal value</th>
<th>-20% nominal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.14</td>
<td>0.12</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>3</td>
<td>0.12</td>
<td>0.94</td>
</tr>
<tr>
<td>4</td>
<td>0.31</td>
<td>0.37</td>
</tr>
<tr>
<td>5</td>
<td>0.52</td>
<td>0.81</td>
</tr>
<tr>
<td>6</td>
<td>0.24</td>
<td>0.20</td>
</tr>
<tr>
<td>7</td>
<td>0.14</td>
<td>0.19</td>
</tr>
<tr>
<td>8</td>
<td>0.20</td>
<td>0.22</td>
</tr>
</tbody>
</table>
Figure 1
Figure 2
Figure 3
Figure 4
Figure 5
Figure 6
Figure 7
Figure 8